Swami Ramanand Teerth Marathwada University, Nanded, Maharashtra, India

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"Study of Different Analytical Methods of Fractional Differential Equations to Solve Mathematical Models"

Research proposal submitted

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In the subject

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I. Introduction to proposed research topic:

This Synopsis gives a brief description of the literature carried out towards execution of the research work. In the subsequent part, we have mentioned the review process, the objective outlines, purpose of carrying area and the problem statement of our study.

Fractional calculus is the theory of integrals and derivatives of arbitrary real order and was first suggested in works by mathematicians such as Leibniz, L'Hôpital, Abel, Liouville, Riemann, etc. The importance of fractional derivatives for modeling phenomena in different branches of science and engineering is due to their nonlocality nature, an intrinsic property of many complex systems. Unlike the derivative of integer order, fractional derivatives do not take into account only local characteristics of the dynamics but considers the global evolution of the system; for that reason, when dealing with certain phenomena, they provide more accurate models of real-world behavior than standard derivatives.

In recent years, fractional order calculus has been studied as alternative calculus in mathematics and it was applied in various field in science and engineering. Numerical techniques are used intensively, recently the Adomian decomposition method, conformable derivative method and variational iterative methods, differential transform method have been used for solving wide range of problems. The use of fractional differentiation for the mathematical modeling of real word physical problems has been widespread in recent years, for example, modeling of earthquake. Now a days fractional calculus is widely used in medical science likely to study some aspects of covid-19 and many other diseases. Applications of fractional derivatives in other fields and related mathematical tools and techniques could be found. In fact, real world processes generally or most likely are fractional order systems.

Definition of the Riemann Liouville Fractional Derivative:

The Riemann-Liouville definition[1] of fractional derivative is as follows:

For $\alpha \in [n-1, n)$, the α-derivative of f is

$$D_a^{\alpha}(f)(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(x)}{(t-x)^{\alpha-n+1}} dx$$

Definition of the Caputo's Fractional Derivative:

Caputo definition[1] of fractional derivative is as follows:.

For $\alpha \in [n-1, n)$, the α derivative of f is

$$D_a^{\alpha}(f)(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^{\alpha} \frac{f^{(n)}(x)}{(t-x)^{\alpha-n+1}} d\text{x}$$

Some Important Methods:

1) Differential Transform Method [2].

2) Conformable Fractional Derivative [3].

3) Variational Iteration Method [4].

4) Adomian Decomposition Method [5].

II. Survey or Review of Literature

Buvaneshwari K. (2021): The author[6], studied that the existence of solution for a coupled system of nonlinear hybrid fractional differential equations involving hadamard derivative with nonlocal boundary conditions.

Ahmed Murshed Kareem (2019): Ahmed Murshed Kareem[3], discussed two different definitions of fractional derivatives, namely Riemann-Liouville and Caputo derivatives, and some basic properties of these derivatives are discussed, Conformable fractional derivatives are addressed. The author focuses on the condition needed in order to guarantee the general solution when we have the particular solution by applied some theorems.

B. Shiri, D. Baleanu (2019) : One of the important classes of coupled systems of algebraic, differential and fractional differential equations (CSADFDEs) is fractional differential algebraic equations (FDAEs). The main difference of such systems with other class of CSADFDEs is that their singularity remains constant in an interval. However, complete classifying and analysing of these systems relay mainly to the concept of the index which can be found in [7].

Singla, Komal (2017): The authors[8], focus on various extensions for the symmetry analysis of nonlinear system of FDEs. The aim of author is to extend the symmetry approach in order to apply them to wider class of FDEs including time fractional nonlinear systems, higher dimensional nonlinear system and variable coefficient nonlinear systems.

V. Prathiban (2015): The V. Prathiban[9], analyse the solutions of fractional partial differential equations. The well-known fractional partial differential equations such as integrodifferential equation, fractional diffusion-wave equation, fractional Burger's equation and system of fractional partial differential equations are considered. Results are analyse and accuracy of solutions are compared.

Vedat Suat Erturk, Shaher Momani (2008): The Vedat Suat Erturk et.al.[2], presents approximate analytical solutions for systems of fractional differential equations using differential transform method. The fractional derivatives are described in the caputo sense.

Ji-Huan He (1999): The homotopy perturbation technique does not depend upon a small parameter in the equation which can be found in [10]. By the homotopy technique in topology, a homotopy is constructed with an imbedding parameter $p \in [0, 1]$ which is considered as a "small parameter". Some examples are given. The approximations obtained by the proposed method are uniformly valid not only for small parameters, but also for very large parameters.

Recently, a new iterative method was presented by Daftardar – Gejji and Jafari[11]. This technique solves many types of nonlinear equations such as ordinary and partial differential equations of integer and fractional order. Also, lots of work have been done by many research workers in relevant area namely as Mistry L., Chand Pal S., Gupta V. G. and Kumar P., B. Ahmad and S. Shivasundaram, S. Z. Rida and A. A. M. Arafa, Aghili A. and Masom M. R.,

Sontakke B. R. and Shaikh S, Aalam, Suneel Kumar, Sunny Chauhan and Pant. S.P. Acharya and many more...

III. Objectives of Proposed Research Work:

1) To acquire the basic knowledge of fractional calculus.
2) To study the solvability of system of fractional differential equations.
3) To develop new results or techniques and study their application in research area.
4) To find out applications of obtained results.
5) Try to apply the applications of fractional differential equation in other branches of science and real life.
6) Try to solve different Mathematical models by different methods.

The proposed work is to prove that how mathematics is useful in other branches of science. We hope that our research work will be useful in applied sciences and medical field.

IV. Research Methodology:

The following steps will be including in the methodology.

1) To refer the relevant reference books on differential equation and fractional differential equation.
2) To study MATLAB, Python, Latex softwares.
3) To study different methods to solve the system of fractional differential equations.
4) To discuss the solvability of system of fractional differential equations.
5) To develop new techniques to solve the system of fractional differential equation.
6) To study the nature of solution of system of fractional differential equation.
7) Computation and analysis of solution of system of fractional differential equations numerically and graphically using different mathematical softwares.

V. Importance of the Study:

Fractional calculus is becoming a useful and key tool in the analysis of scientific problems in Physics, Engineering, Biology and Medical Science, Fractional formalism to be useful in solving practical problems, fractional calculus, integration and differentiation of an arbitrary or fractional order, provides new tools that expand the descriptive power of calculus beyond the familiar integer-order concept of rate of change and area under the curve.

a) International Status: Many scientists and researchers many more contributed in the development of fractional differential equations and Fractional calculus like, Leibniz, Euler, Lagrance, Laplace, Fourier, Abel, Liouvielli, Grunwald, Riemann, Riesz, Miller Ross, Igor Prodlubny, Richtmeyer, Morton, Smith, Anderson, Roache, Lynn R. Willams etc..

b) National Status: Indian's also contributed remarkable in the development of fractional calculus like R. K. Sexana, V. Srivastava, K. N. Rai, S. Das, K. Gupta, S. Saha Roy, V. B. L. Chawrasia, D. Kumar, Pradeep Kumar, D. Bahuguna, Aalam, Suneel Kumar, Sunny Chauhan and Pant. S.P. Acharya worked on some results on fixed point in uniform space. S. L. Sing (Haridwar), R. P. Pant (Nainital), S. Bhalekar, V. Daftardar- Gejji from Maharashtra, D.N. Chaliszar (Gujrat).

c) Scope of Study:

1) Different Mathematical models are constructed in the form of system of fractional differential equations and we will try to study nature of solution of these models using different methods and mathematical softwares.
2) In the area of fractional calculus number of researcher have been developed the results, methods, operators, transformations. We will try to use the application of these in any other branches of science.
3) There are number of methods and alternative definitions, desire approaches to solve the fractional differential equations so there is good scope of analyze these and use it in different fields.

VI. Purpose of carrying out proposed research work:

Differential equations play a major role in Mathematics, Physics and Engineering. The purpose of the research will study different iterative methods of fractional differential equations to obtain their solutions. Also, we will represent graphically. The solution of fractional differential equations obtains by new methods using mathematical softwares Mathematica, MATLAB, Python, Scilab, Mathcad.

Also we will study different methods to obtain the solutions of system of fractional differential equations, such as conformable fractional derivative methods. We will compare the solutions obtain by Adomian Decomposition methods and differential transform methods.

- **Year wise plan of work and targets to be achieved:**

Sr. No	Period	Plan of work	Target to be Achieved
1	First year	1) Study the revelent literature regarding to research topic. 2) To study different methods.	Excellent understanding of the subject and well-organized course work
2	Second year	1) Collecting and analyzing research articles. 2) Presenting research papers in international conferences.	Attempt to obtain fresh results based on existing work and submit some papers.
3	Third year	Publishing some papers and submitting the thesis.	Some more papers and thesis submission.

VII. References:

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